

Classification of totally umbilical slant submanifolds of a Kenmotsu manifold

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Abstract

The purpose of this paper is to classify totally umbilical slant submanifolds of a Kenmotsu manifold. We prove that a totally umbilical slant submanifold M of a Kenmotsu manifold \bar{M} is either invariant or anti-invariant or $\dim M = 1$ or the mean curvature vector H of M lies in the invariant normal subbundle. Moreover, we find with an example that every totally umbilical proper slant submanifold is totally geodesic.

1 Introduction

Slant submanifolds of an almost Hermitian manifold were defined by Chen as a natural generalization of both holomorphic and totally real submanifolds [6]. On the other hand, A. Lotta [15] has introduced the notion of slant immersions into almost contact metric manifolds and obtained the results of fundamental importance. He has also studied the intrinsic geometry of 3-dimensional non anti-invariant slant submanifolds of K -contact manifolds [16]. Later on, Cabrerizo et. al [3] studied the geometry of slant submanifolds in more specialized settings of K -contact and Sasakian manifolds and obtained many interesting results.

On the other hand, in 1954, J.A. Schouten studied the totally umbilical submanifolds and proved that every totally umbilical submanifold of $\dim \geq 4$ in a conformally flat space is conformally flat [17]. After that many authors studied the geometrical aspects of these submanifolds in different settings, including those of [1, 4, 5, 8, 9, 18]. In this paper, we consider M , a totally umbilical slant submanifold tangent to the structure vector field ξ of a Kenmotsu manifold \bar{M} and obtain a classification result that either (i) M is anti-invariant or (ii) $\dim M = 1$ or (iii) $H \in \Gamma(\mu)$, where μ is the invariant normal subbundle under ϕ . We also prove that every totally umbilical proper slant submanifold is totally geodesic. To, this end, we provide an example to justify our results.

2 Preliminaries

A $(2n + 1)$ -dimensional manifold (\bar{M}, g) is said to be an *almost contact metric manifold* if it admits an endomorphism ϕ of its tangent bundle $T\bar{M}$, a vector field ξ , called *structure vector field* and η , the dual 1-form of ξ satisfying the following [2]:

$$\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi(\xi) = 0, \quad \eta \circ \phi = 0 \quad (2.1)$$

and

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad \eta(X) = g(X, \xi) \quad (2.2)$$

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for any X, Y tangent to \bar{M} . An almost contact metric manifold is known to be *Kenmotsu manifold* [12] if

$$(\bar{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X \quad (2.3)$$

and

$$\bar{\nabla}_X \xi = X - \eta(X)\xi \quad (2.4)$$

for any vector fields X, Y on \bar{M} , where $\bar{\nabla}$ denotes the Riemannian connection with respect to g .

Now, let M be a submanifold of \bar{M} . We will denote by ∇ , the induced Riemannian connection on M and g , the Riemannian metric on \bar{M} as well as the metric induced on M . Let TM and $T^\perp M$ be the Lie algebra of vector fields tangent to M and normal to M , respectively and ∇^\perp the induced connection on $T^\perp M$. Denote by $\mathcal{F}(M)$ the algebra of smooth functions on M and by $\Gamma(TM)$ the $\mathcal{F}(M)$ -module of smooth sections of TM over M . Then the Gauss and Weingarten formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y) \quad (2.5)$$

$$\bar{\nabla}_X N = -A_N X + \nabla_X^\perp N, \quad (2.6)$$

for each $X, Y \in \Gamma(TM)$ and $N \in \Gamma(T^\perp M)$, where h and A_N are the second fundamental form and the shape operator (corresponding to the normal vector field N) respectively for the immersion of M into \bar{M} . They are related as

$$g(h(X, Y), N) = g(A_N X, Y). \quad (2.7)$$

Now, for any $X \in \Gamma(TM)$, we write

$$\phi X = TX + FX, \quad (2.8)$$

where TX and FX are the tangential and normal components of ϕX , respectively. Similarly for any $N \in \Gamma(T^\perp M)$, we have

$$\phi N = tN + fN, \quad (2.9)$$

where tN (resp. fN) is the tangential (resp. normal) component of ϕN .

From (2.1) and (2.8), it is easy to observe that for each $X, Y \in \Gamma(TM)$

$$g(TX, Y) = -g(X, TY). \quad (2.10)$$

The covariant derivatives of the endomorphisms ϕ , T and F are defined respectively as

$$(\bar{\nabla}_X \phi)Y = \bar{\nabla}_X \phi Y - \phi \bar{\nabla}_X Y, \quad \forall X, Y \in \Gamma(T\bar{M}) \quad (2.11)$$

$$(\bar{\nabla}_X T)Y = \nabla_X TY - T \nabla_X Y, \quad \forall X, Y \in \Gamma(TM) \quad (2.12)$$

$$(\bar{\nabla}_X F)Y = \nabla_X^\perp FY - F \nabla_X Y \quad \forall X, Y \in \Gamma(TM). \quad (2.13)$$

Throughout, the structure vector field ξ assumed to be tangential to M , otherwise M is simply anti-invariant [15]. For any $X \in \Gamma(TM)$, on using (2.4) and (2.5), we may obtain

$$(a) \quad \nabla_X \xi = X - \eta(X)\xi, \quad (b) \quad h(X, \xi) = 0. \quad (2.14)$$

On using (2.3), (2.5), (2.6), (2.8), (2.9) and (2.11)-(2.13), we obtain

$$(\bar{\nabla}_X T)Y = g(TX, Y)\xi - \eta(Y)TX + A_{FY}X + th(X, Y) \quad (2.15)$$

$$(\bar{\nabla}_X F)Y = fh(X, Y) - h(X, TY) - \eta(Y)FX. \quad (2.16)$$

A submanifold M of an almost contact metric manifold \bar{M} is said to be *totally umbilical* if

$$h(X, Y) = g(X, Y)H, \quad (2.17)$$

where H is the mean curvature vector of M . Furthermore, if $h(X, Y) = 0$, for all $X, Y \in \Gamma(TM)$, then M is said to be *totally geodesic* and if $H = 0$, then M is *minimal* in \bar{M} .

For a totally umbilical submanifold M tangent to the structure vector field ξ of a Kenmotsu manifold \bar{M} , we have

$$g(X, \xi)H = 0, \quad \forall X \in \Gamma(TM). \quad (2.18)$$

There are two possible cases arise, hence we conclude the following:

Case (i): When X and ξ are linearly dependent, i.e., $X = \alpha\xi$, for some non-zero $\alpha \in \mathbb{R}$, then $g(X, \xi) = \alpha$. In this case, from (2.18), we get $H = 0$ with $\dim M = 1$, which is trivial case of totally geodesic submanifold of unit dimension.

Case (ii): When X and ξ are orthogonal, then from (2.18), it is not necessary that $H = 0$, which is the case has to be discussed for totally umbilical submanifolds.

In the following section, we will discuss all possible cases of totally umbilical slant submanifolds.

3 Slant submanifolds

A submanifold M tangent to the structure vector field ξ of an almost contact metric manifold \bar{M} is said to be *slant submanifold* if for any $x \in M$ and $X \in T_x M - \langle \xi \rangle$, the angle between ϕX and $T_x M$ is constant. The constant angle $\theta \in [0, \pi/2]$ is then called *slant angle* of M in \bar{M} . Thus, for a slant submanifold M , the tangent bundle TM is decomposed as

$$TM = D \oplus \langle \xi \rangle$$

where the orthogonal complementary distribution D of $\langle \xi \rangle$ is known as *slant distribution* on M . The normal bundle $T^\perp M$ of M is decomposed as

$$T^\perp M = F(TM) \oplus \mu,$$

where μ is the invariant normal subbundle with respect to ϕ orthogonal to $F(TM)$.

For a proper slant submanifold M of an almost contact metric manifold \bar{M} with the slant angle θ , Lotta [15] proved that

$$T^2 X = -\cos^2 \theta (X - \eta(X)\xi) \quad (3.1)$$

for any $X \in \Gamma(TM)$.

Recently, Cabrerizo et. al [3] extended the above result into a characterization for a slant submanifold in a contact metric manifold. In fact, they have

obtained the following theorem.

Theorem 3.1 [3] *Let M be a submanifold of an almost contact metric manifold \bar{M} such that $\xi \in TM$. Then M is slant if and only if there exists a constant $\lambda \in [0, 1]$ such that*

$$T^2 = \lambda(-I + \eta \otimes \xi). \quad (3.2)$$

Furthermore, in such a case, if θ is slant angle, then it satisfies that $\lambda = \cos^2 \theta$.

Hence, for a slant submanifold M of an almost contact metric manifold \bar{M} , the following relations are consequences of the above theorem.

$$g(TX, TY) = \cos^2 \theta [g(X, Y) - \eta(X)\eta(Y)] \quad (3.3)$$

$$g(FX, FY) = \sin^2 \theta [g(X, Y) - \eta(X)\eta(Y)] \quad (3.4)$$

for any $X, Y \in \Gamma(TM)$.

In the following theorem we consider M as a totally umbilical slant submanifold of a Kenmotsu manifold \bar{M} .

Theorem 3.2 *Let M be a totally umbilical slant submanifold of a Kenmotsu manifold \bar{M} . Then at least one of the following statements is true*

- (i) M is invariant
- (ii) M is anti-invariant
- (iii) M is totally geodesic
- (iv) $\dim M = 1$
- (v) If M is proper slant, then $H \in \Gamma(\mu)$

where H is the mean curvature vector of M .

Proof. As M is totally umbilical slant submanifold, then we have

$$h(TX, TX) = g(TX, TX)H = \cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H.$$

Using (2.5), we obtain

$$\cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H = \bar{\nabla}_{TX}TX - \nabla_{TX}TX.$$

Then from (2.8), we get

$$\cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H = \bar{\nabla}_{TX}\phi X - \bar{\nabla}_{TX}FX - \nabla_{TX}TX.$$

By (2.6) and (2.11), we derive

$$\begin{aligned} \cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H &= (\bar{\nabla}_{TX}\phi)X + \phi\bar{\nabla}_{TX}X + A_{FX}TX \\ &\quad - \nabla_{TX}^\perp FX - \nabla_{TX}TX. \end{aligned}$$

Using (2.3) and (2.5), we obtain

$$\cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H = g(\phi TX, X)\xi - \eta(X)\phi TX + \phi(\nabla_{TX}X + h(X, TX))$$

$$+ A_{FX}TX - \nabla_{TX}^\perp FX - \nabla_{TX}TX.$$

From (2.8), (2.10), (2.17) and the fact that X and TX are orthogonal vector fields on M , we arrive at

$$\begin{aligned} \cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H &= -g(TX, TX)\xi - \eta(X)T^2X - \eta(X)FTX + T\nabla_{TX}X \\ &\quad + F\nabla_{TX}X + A_{FX}TX - \nabla_{TX}^\perp FX - \nabla_{TX}TX. \end{aligned}$$

Then, using (3.2) and (3.3), we get

$$\begin{aligned} \cos^2 \theta \{\|X\|^2 - \eta^2(X)\}H &= -\cos^2 \theta \{\|X\|^2 - \eta^2(X)\}\xi - \cos^2 \theta \eta(X)\{-X + \eta(X)\xi\} \\ &\quad - \eta(X)FTX + T\nabla_{TX}X + F\nabla_{TX}X \\ &\quad + A_{FX}TX - \nabla_{TX}^\perp FX - \nabla_{TX}TX. \end{aligned} \quad (3.5)$$

Taking the inner product with TX in (3.5), for any $X \in \Gamma(TM)$, we obtain

$$0 = g(T\nabla_{TX}X, TX) + g(A_{FX}TX, TX) - g(\nabla_{TX}TX, TX). \quad (3.6)$$

Now, we compute the first and last term of (3.6) as follows

$$g(T\nabla_{TX}X, TX) = \cos^2 \theta \{g(\nabla_{TX}X, X) - \eta(X)g(\nabla_{TX}X, \xi)\}. \quad (3.7)$$

Also, we have

$$g(\nabla_{TX}TX, TX) = g(\bar{\nabla}_{TX}TX, TX).$$

Using the property of Riemannian connection the above equation will be

$$g(\nabla_{TX}TX, TX) = \frac{1}{2}TXg(TX, TX) = \frac{1}{2}TX\{\cos^2 \theta(g(X, X) - \eta(X)\eta(X))\}.$$

Again by the property of Riemannian connection, we derive

$$\begin{aligned} g(\nabla_{TX}TX, TX) &= \cos^2 \theta \{g(\bar{\nabla}_{TX}X, X) - \eta(X)g(\bar{\nabla}_{TX}X, \xi)\} \\ &\quad - \cos^2 \theta \eta(X)g(\bar{\nabla}_{TX}\xi, X). \end{aligned} \quad (3.8)$$

Using (2.4) and the fact that X and TX are orthogonal vector fields on M , the last term of (3.8) is identically zero, then by (2.5), we obtain

$$g(\nabla_{TX}TX, TX) = \cos^2 \theta \{g(\nabla_{TX}X, X) - \eta(X)g(\nabla_{TX}X, \xi)\}. \quad (3.9)$$

Thus, from (3.7) and (3.9), we get

$$g(T\nabla_{TX}X, TX) = g(\nabla_{TX}TX, TX). \quad (3.10)$$

Using this fact in (3.6), we obtain

$$0 = g(A_{FX}X, TX) = g(h(TX, TX), FX).$$

As M is totally umbilical slant, then from (2.17) and (3.3), we get

$$0 = \cos^2 \theta \{\|X\|^2 - \eta^2(X)\}g(H, FX). \quad (3.11)$$

Thus, from (3.11), we conclude that either $\theta = \pi/2$, that is M is anti-invariant which part (ii) or the vector field X is parallel to the structure vector field

ξ , i.e., M is 1-dimensional submanifold which is fourth part of the theorem or $H \perp FX$, for all $X \in \Gamma(TM)$, i.e., $H \in \Gamma(\mu)$ which is the last part of the thorem or $H = 0$, i.e., M is totally geodesic which is (iii) or $FX = 0$, $\forall X \in \Gamma(TM)$, i.e., M is invariant which is part (i). This proves the theorem completely. ■

Now, if we consider M , a proper slant submanifold of a Kenmotsu manifold \bar{M} , then neither M is invariant nor anti-invariant (by definition of proper slant) and also neither $\dim M = 1$. Hence, by the above result, only possibility is that $H \in \Gamma(\mu)$ for a totally umbilical proper slant submanifold. Thus, we prove the following main result.

Theorem 3.3 *Every totally umbilical proper slant submanifold of a Kenmotsu manifold is totally geodesic.*

Proof. Let M be a totally umbilical proper slant submanifold of a Kenmotsu manifold \bar{M} , then for any $X, Y \in \Gamma(TM)$, we have

$$\bar{\nabla}_X \phi Y - \phi \bar{\nabla}_X Y = g(\phi X, Y)\xi - \eta(Y)\phi X.$$

From (2.5) and (2.8), we obtain

$$\bar{\nabla}_X TY + \bar{\nabla}_X FY - \phi(\nabla_X Y + h(X, Y)) = g(TX, Y)\xi - \eta(Y)TX - \eta(Y)FX.$$

Again using (2.5), (2.6) and (2.8), we get

$$\begin{aligned} g(TX, Y)\xi - \eta(Y)TX - \eta(Y)FX &= \nabla_X TY + h(X, TY) - A_{FY}X \\ &\quad + \nabla_X^\perp FY - T\nabla_X Y - F\nabla_X Y - \phi h(X, Y). \end{aligned}$$

As M is totally umbilical, then

$$\begin{aligned} g(TX, Y)\xi - \eta(Y)TX - \eta(Y)FX &= \nabla_X TY + g(X, TY)H - A_{FY}X + \nabla_X^\perp FY \\ &\quad - T\nabla_X Y - F\nabla_X Y - g(X, Y)\phi H. \end{aligned} \quad (2.12)$$

Taking the inner product with ϕH in (3.12) and using the fact that $H \in \Gamma(\mu)$, we obtain

$$g(\nabla_X^\perp FY, \phi H) = g(X, Y)\|H\|^2.$$

Using (2.6) and the property of Riemannian connection, the above equation takes the form

$$g(FY, \nabla_X^\perp \phi H) = -g(X, Y)\|H\|^2. \quad (3.13)$$

Now, for any $X \in \Gamma(TM)$, we have

$$\bar{\nabla}_X \phi H = (\bar{\nabla}_X \phi)H + \phi \bar{\nabla}_X H.$$

Using (2.3), (2.6), (2.8) and the fact that $H \in \Gamma(\mu)$, we obtain

$$-A_{\phi H}X + \nabla_X^\perp \phi H = -TA_HX - FA_HX + \phi \nabla_X^\perp H. \quad (3.14)$$

Also, for any $X \in \Gamma(TM)$, we have

$$\begin{aligned} g(\nabla_X^\perp H, FX) &= g(\bar{\nabla}_X H, FX) \\ &= -g(H, \bar{\nabla}_X FX). \end{aligned}$$

Using (2.8), we get

$$g(\nabla_X^\perp H, FX) = -g(H, \bar{\nabla}_X \phi X) + g(H, \bar{\nabla}_X PX).$$

Then from (2.5) and (2.11), we derive

$$g(\nabla_X^\perp H, FX) = -g(H, (\bar{\nabla}_X \phi)X) - g(H, \phi \bar{\nabla}_X X) + g(H, h(X, PX)).$$

Using (2.3) and (2.17), the first and last term of right hand side of the above equation are identically zero and hence by (2.2), the second term gives

$$g(\nabla_X^\perp H, FX) = g(\phi H, \bar{\nabla}_X X).$$

Again, using (2.5) and (2.17), finally we obtain

$$g(\nabla_X^\perp H, FX) = g(\phi H, H)\|X\|^2 = 0.$$

This means that

$$\nabla_X^\perp H \in \Gamma(\mu). \quad (3.15)$$

Now, taking the inner product in (3.14) with FY , for any $Y \in \Gamma(TM)$, we get

$$g(\nabla_X^\perp \phi H, FY) = -g(FA_H X, FY) + g(\phi \nabla_X^\perp H, FY).$$

Using (3.15), the last term of the right hand side of the above equation will be zero and then from (3.4), (3.13), we obtain

$$g(X, Y)\|H\|^2 = \sin^2 \theta \{g(A_H X, Y) - \eta(Y)g(A_H X, \xi)\}. \quad (3.16)$$

Hence, by (2.7) and (2.17), the above equation reduces to

$$g(X, Y)\|H\|^2 = \sin^2 \theta \{g(X, Y)\|H\|^2 - \eta(Y)g(h(X, \xi), H)\}. \quad (3.17)$$

Since, for a Kenmotsu manifold \bar{M} , $h(X, \xi) = 0$, for any X tangent to \bar{M} , thus we obtain

$$g(X, Y)\|H\|^2 = \sin^2 \theta g(X, Y)\|H\|^2.$$

Therefore, the above equation can be written as

$$\cos^2 \theta g(X, Y)\|H\|^2 = 0. \quad (3.18)$$

Since, M is proper slant, thus from (3.18), we conclude that $H = 0$ i.e., M is totally geodesic in \bar{M} . This completes the proof of the theorem. ■

Now, we give the following counter example of totally geodesic submanifold of R^5 .

Example 3.1 Consider a 3-dimensional proper slant submanifold with the slant angle $\theta \in [0, \pi/2]$ of R^5 with its usual Kenmotsu structure

$$x(u, v, t) = 2(u \cos \theta, u \sin \theta, v, 0, t).$$

If we denote by M a slant submanifold, then its tangent space TM span by the vectors

$$e_1 = \frac{\partial}{\partial u} + 2 \cos \theta \left(\frac{\partial}{\partial x^1} + y^1 \frac{\partial}{\partial t} \right) + 2 \sin \theta \left(\frac{\partial}{\partial x^2} + y^2 \frac{\partial}{\partial t} \right),$$

$$e_2 = \frac{\partial}{\partial v} = 2\frac{\partial}{\partial y^1}, \quad e_3 = \frac{\partial}{\partial t} = \xi.$$

Moreover, the vector fields

$$e_1^* = -2 \sin \theta \left(\frac{\partial}{\partial x^1} + y^1 \frac{\partial}{\partial t} \right) + 2 \cos \theta \left(\frac{\partial}{\partial x^2} + y^2 \frac{\partial}{\partial t} \right),$$

$$e_2^* = 2 \frac{\partial}{\partial y^2}$$

form the basis of $T^\perp M$. Furthermore, using Koszul's formula, we get $\bar{\nabla}_{e_i} e_i = -e_3 = -\xi$, $i = 1, 2$ and when $i \neq j$, then $\bar{\nabla}_{e_i} e_j = 0$, for $i, j = 1, 2, 3$. Also, $\bar{\nabla}_{e_3} e_3 = 0$, thus, from Gauss formula and (2.14), we obtain

$$h(e_1, e_1) = 0, \quad h(e_2, e_2) = 0, \quad h(e_1, e_2) = 0, \quad h(e_3, e_3) = 0$$

and hence we conclude that M is totally geodesic.

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